Math 330: Introduction to Higher Math, Section 1, Spring 2009 — Quiz # 2, February 20

Name: ....

For each of the following five questions, seven possible answers are provided, but only one of them is correct: circle the corresponding letter!

The symbols X and Y are used below to represent statements (e.g., "These pretzels are making me thirsty"), and similarly the symbol $P(n)$ denotes a property about some natural number n (e.g., "n is even").

1. If you know that X implies Y, then you can also conclude that:
   A] X is true, and Y is also true.
   B] X cannot be false.
   C] Y cannot be false.
   D] At least one of X and Y is true.
   E] If Y is true, then X is true.
   F] If Y is false, then X is false.
   G] If X is false, then Y is false.
2. Which of the following strategies is *not* a valid way to show that "X implies Y"?
   A] Assume that X is true, and then use this to show that Y is true.
   B] Assume that Y is false, and then use this to show that X is false.
   C] Show that either X is false, or Y is true, or both.
   D] Assume that X is true and Y is false, and deduce a contradiction.
   E] Assume that X is false and Y is true, and deduce a contradiction.
   F] Show that X implies some intermediate statement Z, and then show that Z implies Y.
   G] Show that some intermediate statement Z implies Y, and then show that X implies Z.
3. If you want to *disprove* the claim that "X implies Y", you need to show* that:
   A] Y is true, but X is false.
   B] X is true, but Y is false.
   C] X is false.
   D] Y is false.
   E] Both X and Y are false.
   F] Exactly one of X and Y is false.
   G] At least one of X and Y is false.
4. Which of the following strategies is *not* a valid way to show that "For all $n \in \mathbb{N}$, $P(n)$ is true"?
   A] Assume there exists an $n \in \mathbb{N}$ for which $P(n)$ is false, and derive a contradiction.
   B] Let $n \in \mathbb{N}$ be an arbitrary natural number. Show that $P(n)$ is true.
   C] Show that for any $k \in \mathbb{N}$, if $P(k)$ is true then $P(k+1)$ is true, and then show that $P(1)$ is true.
   D] Show that $P(1)$ is true, and then show that for any $k \in \mathbb{N}$, if $P(k)$ is true then $P(k+1)$ is true.
   E] Show that $P(1)$ is true, and then show that for any $k \in \mathbb{N}$, if $P(k)$ is false then $P(k+1)$ is false.
   F] Show that $P(1)$ is true, and then show that for any $k \in \mathbb{N}$, if $P(k+1)$ is false then $P(k)$ is false.
   G] Show that $P(1)$ is true. Assume $\exists\, k \in \mathbb{N}$ s.t. $P(k)$ is true but $P(k+1)$ is false, and derive a contradiction.
5. If you want to *disprove* the claim that "For some $n \in \mathbb{N}$, $P(n)$ is true", you need to:
   A] Assume that for all $n \in \mathbb{N}$, $P(n)$ is true, and derive a contradiction.
   B] Let $n \in \mathbb{N}$ be an arbitrary natural number. Show that $P(n)$ is false.
   C] Show that there exists an $n \in \mathbb{N}$ for which $P(n)$ is false.
   D] Disprove that for any $k \in \mathbb{N}$, if $P(k)$ is true then $P(k+1)$ is true.
   E] Show that $P(1)$ is false, and then disprove that for any $k \in \mathbb{N}$, if $P(k)$ is true then $P(k+1)$ is true.
   F] Show that $P(1)$ is false, and then show that for any $k \in \mathbb{N}$, if $P(k+1)$ is false then $P(k)$ is false.
   G] Show that $P(n)$ being true does not necessarily imply that n is a natural number.

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*Beware of the difference between "you need to do ..." and "in certain cases, but not in general, it would be enough to do ..."!

Adapted from a quiz by Terence Tao at `http://scherk.pbwiki.com/`