Math 330: Introduction to Higher Math, Section 1, Spring 2009 — Quiz # 1, February 5

Name: ....

For each of the following ten questions, seven possible answers are provided, but only one of them is correct: circle the corresponding letter!

The symbols X, Y, and Z are used below to represent statements (e.g., "These pretzels are making me thirsty"); the symbol $P(n)$ denotes a property about some integer n (e.g., "n is odd"), and similarly $Q(n, m)$ denotes a property about two integers n and m (e.g., "n is divisible by m").

1. If you know that X implies Y, then you can also conclude that:
   A] X is true, and Y is also true.
   B] X cannot be false.
   C] Y cannot be false.
   D] At least one of X and Y is true.
   E] If Y is true, then X is true.
   F] If Y is false, then X is false.
   G] If X is false, then Y is false.
2. Which of the following strategies is *not* a valid way to show that "X implies Y"?
   A] Assume that X is true, and then use this to show that Y is true.
   B] Assume that Y is false, and then use this to show that X is false.
   C] Show that either X is false, or Y is true, or both.
   D] Assume that X is true and Y is false, and deduce a contradiction.
   E] Assume that X is false and Y is true, and deduce a contradiction.
   F] Show that X implies some intermediate statement Z, and then show that Z implies Y.
   G] Show that some intermediate statement Z implies Y, and then show that X implies Z.
3. If you want to *disprove* the claim that "X implies Y", you need to show* that:
   A] Y is true, but X is false.
   B] X is true, but Y is false.
   C] X is false.
   D] Y is false.
   E] Both X and Y are false.
   F] Exactly one of X and Y is false.
   G] At least one of X and Y is false.
4. If you want to *disprove* the claim that "Both X and Y are true", you need to show that:
   A] X does not imply Y, and Y does not imply X.
   B] X is true if and only if Y is false.
   C] X is false.
   D] Y is false.
   E] Both X and Y are false.
   F] Exactly one of X and Y is false.
   G] At least one of X and Y is false.
5. If you want to *disprove* the claim that "At least one of X and Y is true", you need to show that:
   A] X does not imply Y, and Y does not imply X.
   B] X is true if and only if Y is false.
   C] X is false.
   D] Y is false.
   E] Both X and Y are false.
   F] Exactly one of X and Y is false.
   G] At least one of X and Y is false.

*Beware of the difference between "you need to do ..." and "in certain cases, but not in general, it would be enough to do ..."!

6. If you want to *disprove* the claim that "For all integers n, $P(n)$ is true", you need to:
   A] Show that there exists an integer n for which $P(n)$ is false.
   B] Show that there exists an n which is not an integer, but for which $P(n)$ is still true.
   C] Show that for all integers n, $P(n)$ is false.
   D] Show that for all integers n, $P(n)$ is true.
   E] Show that $P(n)$ being true does not necessarily imply that n is an integer.
   F] Assume there exists an integer n for which $P(n)$ is true, and derive a contradiction.
   G] Show that for every integer n, there exists an integer m not equal to n for which $P(m)$ is true.

7. If you want to *disprove* the claim that "For some integer n, $P(n)$ is true", you need to:
   A] Show that there exists an integer n for which $P(n)$ is false.
   B] Show that there exists an n which is not an integer, but for which $P(n)$ is still true.
   C] Show that for all integers n, $P(n)$ is false.
   D] Show that for all integers n, $P(n)$ is true.
   E] Show that $P(n)$ being true does not necessarily imply that n is an integer.
   F] Assume that for every integer n, $P(n)$ is true, and derive a contradiction.
   G] Show that for every integer n, there exists an integer m not equal to n for which $P(m)$ is true.

8. If you want to prove the claim that "For every integer n, there exists an integer m such that $Q(n, m)$ is true", you need to do the following:
   A] Let n and m be arbitrary integers. Then show that $Q(n, m)$ is true.
   B] Find an integer n and an integer m such that $Q(n, m)$ is true.
   C] Let n be an arbitrary integer. Then find an integer m (possibly depending on n) such that $Q(n, m)$ is true.
   D] Let m be an arbitrary integer. Then find an integer n (possibly depending on m) such that $Q(n, m)$ is true.
   E] Find an integer n such that $Q(n, m)$ is true for every integer m.
   F] Find an integer m such that $Q(n, m)$ is true for every integer n.
   G] Show that whenever $Q(n, m)$ is true, then n and m are integers.

9. If you want to *disprove* the claim that "For every integer n, there exists an integer m such that $Q(n, m)$ is true", you need to show that:
   A] There exists an integer n such that for all integers m, $Q(n, m)$ is false.
   B] There exist integers n and m such that $Q(n, m)$ is false.
   C] For every integer n and every integer m, $Q(n, m)$ is false.
   D] For every integer n, there exists an integer m such that $Q(n, m)$ is false.
   E] For every integer m, there exists an integer n such that $Q(n, m)$ is false.
   F] There exists an integer m such that for all integers n, $Q(n, m)$ is false.
   G] If $Q(n, m)$ is true, then n and m are not integers.

10. If you want to *disprove* the claim that "There exists an integer n such that for all integers m, $Q(n, m)$ is true", you need to show that:
   A] There exists an integer n such that for all integers m, $Q(n, m)$ is false.
   B] There exist integers n and m such that $Q(n, m)$ is false.
   C] For every integer n and every integer m, $Q(n, m)$ is false.
   D] For every integer n, there exists an integer m such that $Q(n, m)$ is false.
   E] For every integer m, there exists an integer n such that $Q(n, m)$ is false.
   F] There exists an integer m such that for all integers n, $Q(n, m)$ is false.
   G] If $Q(n, m)$ is true, then n and m are not integers.

Adapted from a quiz by Terence Tao at `http://scherk.pbwiki.com/`.